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ORIGINAL PAPER

# What is the therapeutically active ingredient of homeopathic potencies ?

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**The nature of the 'active ingredient', in homeopathic high dilutions is investigated. A model for every degree of dilution is introduced; within this the active ingredient can be dealt with in physical terms. In mathematical terms this model has features which correspond to the axioms of weak quantum theory. Features which are similar to entanglement in ordinary quantum theory are discussed in particular.** *Homeopathy* (2003) **92,** 145–151.

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## Introduction

Research on homeopathic high dilutions (high potencies) can roughly be classified into three topics. The first concerns with the practice of homoeopathy and therefore includes research on clinical practice, provings, case reports, etc.

The second topic includes statistically based research work on high potencies, for instance the methodology of provings and other clinical studies, the realization of trials and the statistical interpretation of data. Methodology is faced with the problem of developing designs which refute the widespread view that testing high potencies in placebo-controlled trials is a test of one placebo against another. This problem would be much easier to deal with if the nature of the 'active ingredient' in high potencies were known. The third research topic therefore consists of the totality of attempts to get this knowledge.

This paper is concerned with a working hypothesis about the nature of the 'active ingredient' of high potencies and its physical rationale. The hypothesis is that the 'active ingredient' of high potencies consists of states which the remedy as a molecular system can take on and which are very similar to those states in quantum systems which are correlated to each other but not causally dependent. The starting point is the presumption (now nearly 50 years old) that nuclear magnetic resonance spectroscopy (NMR) is a suitable method for measuring the 'active ingredient' of high dilutions. At the end of the paper some arguments are listed which specify the possible role of NMR in the investigation of an 'active ingredient'. The basis of all these arguments is the undeniable fact that quantum physics gives us the most accurate general formal picture of nature to date, even if this picture is sometimes counterintuitive. Correlations between particles in quantum systems are of particular interest, showing properties of nature which never have been recognized as correlated in the macroscopic world.

In order to reach the idea that quantum system-like correlations are responsible for the therapeutically active ingredient in homeopathic potencies, potentization is adapted to a thought experiment which allows us to deal with a physically reasonable system at every degree of dilution, even beyond Avogadro's number.

## The therapeutically active ingredient

In 1985 Bergholz[1] gave a physical meaning to the expression 'Therapeutically Active Ingredient' (TAI) of homeopathic potencies, the underlying idea can be traced back to the famous article by Barnard published 20 years earlier.[2] Bergholz is a physicist, the basis of his work was his critical attitude towards NMR

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measurements by Smith, Boericke[3] and others. It seemed reasonable to Bergholz to propose, as a working hypothesis, that a homeopathic potency beyond Avogadro's number must possess some TAI which cannot be ascribed to the atoms or molecules of the original solute, since none remain. In other words, Bergholz demanded a search for a physical structure generated by potentization. He proposed a series of experiments by which one should be able to determine the nature of the TAI. Despite his critical attitude towards the NMR results, he believed NMR to be a promising tool to empirically investigate the TAI. Unfortunately Bergholz's programme has never been carried out in full. NMR effects (for an overview see reference 4), however, have been measured reproducibly by several authors.

Barnard, Smith and Boericke, and subsequently Bergholz's article proposed the idea of a structure transfer triggered by the process of potentization. Furthermore, it was thought that this transfer might be experimentally verified. It was believed that structure transfer is not restricted to potencies below Avogadro's number. At least Bergholz's paper gave the impression that the TAI is more or less identifiable with those forces in homeopathic potencies, which according to Hahnemann, are responsible for 'reharmonizing the vital force'.

However, the problem remains that neither results from NMR experiments nor results from experiments with any other device can be reasonably interpreted as due to a TAI. Recent findings (see references 5 and 6) have even given cause for concern that there could be artifacts lurking in some of the original data, masquerading as real observations. Moreover, to many people it has not been proven, that NMR can actually reproducibly and accurately distinguish between homeopathically potentized solutions and those that have merely undergone serial dilution. One of the reasons for this attitude is a lack of theoretical framework to understand how a TAI might be responsible for the features observed in NMR measurements. Clearly, such a framework would be more acceptable if it were more formally and scientifically based.

## The sequential box model

The sequential box model (SBM) was introduced in 1997 (see reference 7 and Figure 1). The SBM proposes that for theoretical purposes it is possible to keep a constant volume of mother tincture physically present in every subsequent potency. This leads to an appropriate use of physical reasoning when a TAI specific for the mother tincture is sought in high potencies. The SBM can be understood by the following thought experiment.

Imagine a certain volume of mother tincture being present in a box $B_0$. Then imagine the contents of $B_0$ being poured out into another box $B_1$, which is 10 times bigger then $B_0$ and which is already 9/10th full of solvent. Imagine then $B_1$ being vigorously shaken as in

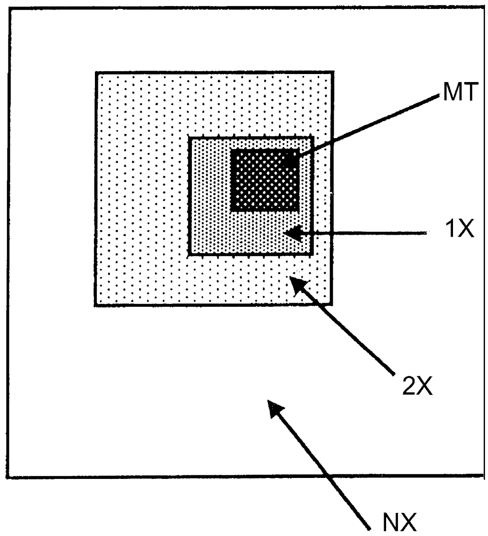


**Figure 1** Diagrammatic representation of the sequential box model. MT = mother tincture, 1X,2X,…,NX = potencies.

the preparation of homeopathic medicines. Imagine then the whole contents of $B_1$ being poured out into a box $B_2$, which is 10 times bigger as $B_1$ and again 9/10th full of solvent.

It can easily be seen that this procedure can be continued stepwise up to an arbitrary potency NX and that the box $B_N$ corresponding to NX contains the whole contents of $B_0$. Therefore for every potency, even for those corresponding to degrees of dilution far beyond Avogadro's number, the problem of looking for a TAI can, at least in principle, be understood in a scientifically reasonable manner. Needless to say, in practice this procedure will come to an end very soon because of the unrealizable dimensions of the boxes. Needless to say, too, that every result on a TAI in this system has to be rescaled to dimensions which correspond to volumes of a remedy.

If $B_N$ is looked upon as a classical system, either the possibility of pairwise interactions or the influence of an external field is necessary for molecules to communicate, for instance in order to build up a structure. This is also true for $B_N$ viewed as a system in which chemical reactions occur. In both cases the probability of finding at least one molecule of the mother tincture in a small subvolume of $B_N$ tends to zero when $N > 23$. This means that even in the SBM an active ingredient in homeopathic potencies is not guaranteed. $B_N$ has to be viewed as possessing some additional properties so that it is not necessary to assume pairwise interactions for particles to communicate with each other. Properties of this kind are called correlations. Correlations of particles which might be responsible for structures are theoretically possible and experimentally demonstrable in quantum systems.

## Correlations in quantum systems

This section gives a short review of correlations in quantum mechanical systems. It is intended to convey the idea that particular features similar to those of correlated quantum systems have to be sought in the SBM or even emerge in the SBM. This review is best done by browsing through the history of investigations into this phenomenon (see for instance references 8–10).

After the discovery of quantum mechanics it was realized that it contains counterintuitive features. These features were the subject of the famous dialogue between Einstein and Bohr. In his 1935 paper[11] with Podolsky and Rosen, commonly cited as the EPR thought experiment, Einstein considered a quantum system consisting of two particles such that, while neither position nor momentum of either particle is well defined, the sum of their positions and the difference of their momenta are both precisely defined. In accordance with the laws of quantum mechanics Einstein came to the conclusion that a measurement of either position or momentum performed on one particle immediately implies a precise position or momentum, respectively, for the other particle, without interaction of the two particles. This correlation remains true when the two particles are separated by an arbitrary distance. For Einstein this example of a proper quantum mechanical system was the proof of the incompleteness of quantum mechanics. Schrödinger named this kind of correlation in quantum systems 'entanglement'.

In reply to Einstein's example, Bohr argued that the two particles in the EPR case are always parts of one quantum system and thus measurement on one particle changes the possible predictions that can be made for the whole system and therefore for the other particle.

In 1951 Bohm[12] reformulated the EPR phenomenon in terms of spin-entangled particles. He considered a system consisting of two particles at rest, the total spin of the system being zero. Then the system is split and the particles are separated by an arbitrary distance. Spin entanglement in this configuration means that the measurement of the spin of one of the particles automatically fixes the spin of the other and destroys the entangled state.

In 1964 Bell[13] showed that, for entangled systems, measurements of correlated quantities should yield different results to those expected if one assumes that the properties of the system measured are present prior to, and independent of, the observation. With Bell's paper it became possible to investigate quantum entanglement experimentally. In 1982 Aspect *et al*[14] were the first to realise such experiments successfully. Since then many applications of quantum entanglement have been developed and research work on the phenomenon has intensified. With respect to a possible application of entanglement-like phenomena to the understanding of the nature of high potencies a few results of this research work are worth mentioning.

It has been shown, in theory, that the number of entangled particles in a system is not necessarily to be restricted to two. Julsgaard *et al*[15] demonstrated experimentally the entanglement of two macroscopic objects, each consisting of a caesium gas sample containing about $10^{12}$ atoms. The entangled spin state could be maintained for 0.5 ms. Altewischer *et al*[16] demonstrated experimentally the survival of entanglement, when the entangled particles passed through a thick metal film. A number of papers describe the use of NMR for investigating phenomena which are closely related to entanglement (for an overview see reference 8). This of course is only reasonable if spin states are expected to be entangled. It does not make sense to use NMR for investigating entanglement-like phenomena for other physical variables for instance position and momentum as in the original EPR-thought experiment.

With respect to entanglement as a general phenomenon intrinsic to nature one might be tempted to suppose that the totality of the particles in a $B_N$ can be looked upon as being brought into an entangled state by potentization. This would mean, that every potency in the SBM differs from all other potencies by a physical quantity. Such reasoning is supported by two arguments. Firstly, the stepwise dilution and the vigorous shaking at each step ensure that all particles in $B_N$ come into intensive contact with each other. Secondly, potentization ensures that most of the pairs or samples of molecules in $B_N$ are separated by distances which are only limited by the dimensions of $B_N$.

But, the experiments of Julsgaard *et al*[15] (see above) and others make it rather improbable that the latter supposition is a description of reality corresponding to a TAI, unless it could be shown that for special qualities of $B_N$ or even for qualities of clusters of particles in $B_N$ the particles referred to had been in an entangled state at sometime. Additionally, it is by no means clear how mechanical succussions could create spin entanglement.

For all these reasons it may be asked, whether or not entanglement in quantum systems is a particular case of a more general class of phenomena in nature. Sometimes these phenomena are called non-local phenomena, meaning the correlation of two or more events at the same time at different space positions. In the following the application of an axiomatized representation of non-locality, not necessarily restricted to spin entanglement, to the high potency problem in the SBM is investigated.

## Quantum theory and 'weak quantum theory'

Quantum theory is the theoretical description of the behavior of quantum systems. Usually its axiomatic formulation is given in terms of Hilbert spaces. This Hilbert space formulation is related to algebraic quantum theory by the so-called Gelfand–Naimark Segal construction. Algebraic quantum theory was used by Atmanspacher *et al*[17] in the formulation of 'weak quantum theory' which is a possible theoretical basis for non-local phenomena to take place in a physically admissible way.

We begin with a short overview of some basic features of algebraic quantum theory (see for instance references 18 and 19) and of the main features of 'weak quantum theory'. Technical details in Appendix A. Quantum systems are described by their states and their observables. Observables, denoted by $A$,$B$,..., are properties of a system which can be measured (at least in principle). The set of possible outcomes of the measurement of an observable is a subset of the complex numbers. In algebraic quantum theory the totality of mathematical objects belonging to the observables of a quantum system generates an algebraic structure for operators, a so-called $C^*$-Algebra. Because of this structure addition and multiplication of operators as well as multiplication of operators by complex numbers are defined. Henceforth we will not explicitely distinguish between observables and the mathematical objects, the operators assigned to them.

Quantum systems can be in different physical states, here denoted by $z$, $z_1$,.... For formal reasons it is assumed that the impossible state '$z = 0$' is also a member of the set of states. States can be pure or composite. Weyl[20] identified the pure states of a quantum system with the extreme points of a convex set through which enabled him, using a theorem proved by Minkowsky (see reference 21), to show, that composite quantum states are always convex combinations of pure ones. This is the background to one of the most essential differences between quantum theory and 'weak quantum theory'. In quantum theory the set of all states is a convex set which means that for every two states $z_1$ and $z_2$, their convex combination

$$z = \lambda z_1 + (1 - \lambda) z_2$$

(where the $\lambda$ are arbitrary numbers between 0 and 1) is also a state. For 'weak quantum theory' (see below) no addition of observables is specified and the set of states cannot be presupposed to be convex. This means that one of the most important properties of a state (pure or composite) depends on the existence of a '+'-operation. As the result of a measurement necessarily is a real number, the set of measurable outcomes of an observable $A$ is restricted. This is equivalent to the use of self-adjoint operators $A^*$ only.

The conceptually simplest observables are propositions, measurements which have only two alternative outcomes, often called 'yes' and 'no', the value '1' is assigned to the 'yes' answer and '0' to 'no'. Mathematically propositions are represented by self-adjoint projection operators, operators $P$ for which $P^2 = P$.

In quantum theory the algebra of observables is generally not commutative. This means that if $A$ and $B$ are observables, in general

$$A(B(z)) \neq B(A(z))$$

or

$$AB - BA \neq 0$$

$AB - BA$ is called the commutation relation between $A$ and $B$.

In 2002 Atmanspacher *et al*[17] gave a set of assumptions under which particular features of quantum theory can be generalized to a framework broader than ordinary quantum theory. Essentially the authors specify a structured set of minimal requirements for a meaningful general theory of observables and states of a system. These requirements are given as axioms (see Appendix A). Between quantum theory and the theory of Atmanspacher *et al* there are intermediate theories, obtained by modifying the axioms stepwise. Therefore, they called their theory 'weak quantum theory'.[17]

For the purposes of this paper, this following features of weak quantum theory are important:

- Characteristically in 'weak quantum theory' there is no quality like Planck's constant ($h$). As a consequence the commutation relations $|AB - BA| \geq h$ must be replaced symbolically by $|AB - BA| \geq$ ? so that entanglement is not restricted to a particular degree of non-commutativity of observables.
- Addition of observables and therefore linear combination of states are not defined. Therefore, no representation of mixed states as convex linear combinations of pure states can be given.
- Bell's inequality cannot be generalized to 'weak quantum theory'.
- Incompatibility and complementarity arise, as in ordinary quantum theory, due to the non-commutativity of the multiplication of observables.
- Entanglement arises if, for a composite system, observables pertaining to the whole system are incompatible with observables of its parts. This latter quality corresponds to Bohr's argument mentioned above.

## 'Weak quantum theory' and the TAI problem

To ensure that specified features of a system $\Sigma$ behave according to rules isomorphic to those of another system $B$, it is necessary:

- to know the axiomatic description of $B$,
- to formalize properties of the specified features of $\Sigma$,
- to map the features of $\Sigma$ (mathematically) onto the axioms of $B$.

In our case we want to show that the SBM (system $\Sigma$) can be looked upon as a system in which weak quantum theory is possible (system $B$), therefore we have to:

- know the axioms of 'weak quantum theory',
- formulate the specifications for the SBM,

- show that $\Sigma$ can be mapped onto the axioms of 'weak quantum theory'.

Let the system $\Sigma$ be identified with the contents of $B_N$ for a fixed $N > 0$ and let the single particles (atoms or molecules) in $B_N$ be the subsystems of $\Sigma$ (see also Appendix A). In $\Sigma$ let the notion of a spin-like quality as a measure of complexity be defined. In the simplest case this is done by looking at couplings of $k(k = 3, 4, \ldots)$ spins of particles instead of only pairs of spins. Let the set of observables $A$ be defined as the set of all continuous deformations of spin configurations. As described in Appendix B, it can be shown that the boxes of the SBM together with these specifications satisfy the axioms of 'weak quantum theory' (see also Appendix A).

Something additional must be said about the logical observables in axiom VI (see Appendix A). As the spin configurations considered here are formally very similar, for instance, to systems which are able to store and to recover information (neural nets, see for instance reference 22) as well as to systems which might be investigated by NMR (see for instance reference 23), the meaning of a spin variable has to be generalized in order to identify a TAI in our model system (the SBM). This generalization will strongly influence the definition of logical observables.

As summarized in Appendix B the axioms of 'weak quantum theory' are satisfied by our definition of systems in the SBM with observables describing the complexity of the ordering of the spins. Now the question arises, whether there is a particular sense in which these spin configurations are stable in time. Since according to weak quantum theory the observables are not commutative in general we have

$$(AB - BA) = k \geq \varepsilon$$

where $A$,$B$ are arbitrary observables and $k$ and $\varepsilon$ are positive but arbitrary numbers. In analogy to weak stationarity in Euclidean theories we recommend the following formal expression for the recognition of a stability in time:

$$(AB - BA)_t = (AB - BA)_{t+\Delta t} = k \geq \varepsilon$$

for numbers $\Delta t > 0$, where $(AB - BA)_t$ means non-commutativity at time $t$.

We have now shown how the SBM subsystems can be defined so that for every particular box $B_N$ a structure is present. These systems can also be enriched (at least theoretically) by the definition of stability in time. But nothing has been said about the transfer of the structure when potentization is applied to the contents of $B_N$ in order to produce the contents of $B_{N+1}$. Obviously $B_N$ and $B_{N+1}$ have to be looked upon as stable but different systems. This means that potentization has to be something like a phase transition. In terms of the stability of the different $B_N$'s a phase transition means a change of the topology of the structure of the $B_N$ when potentized to spin system $B_{N+1}$.

## Outlook

I have shown that the SBM together with some additional assumptions on the couplings of spin-like variables satisfies the prerequisites of a system in which non-locality is possible. This property strongly depends on the assumed couplings and variables. As a consequence which couplings and variables, of the variety allowed in Appendix B, are realisable from an experimental point of view and needs to be investigated. But nothing in the model is so strongly related to spins in the quantum-mechanical sense that it could not be changed to a quantity which only has its mathematical handling in common with real spin. Therefore, the situation is rather speculative and needs a lot of further systematic work. Even experimentally the existence of general non-local phenomena is not fully proven. Probably Bohr's answer to Einstein's example, mentioned above, gives a good starting point for systematic research.

As an experimental starting point it should be determined under which conditions particles or groups of particles in homeopathic potencies, distinguished from the solvent, are correlated by their spins. In the case of real spin entanglement this presumably can best be done by NMR.

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## Appendix A. The axioms of weak quantum theory

Atmanspacher *et al* consider any part of reality in a general sense as a system ($\Sigma$). They take any meaningful property of the system which can be investigated as observables ($A$,$B$…). As the states ($Z$) of the system they take various degrees of knowledge about the system. As in ordinary quantum theory a set of observables and a set of (nontrivial) states is associated with every system.

The list of axioms is given in slightly modified from as follows:

I. There are sets spec $A$, spec $B$, …. belonging to the observables and consisting of the possible outcomes of measurements of $A$,$B$,…
II. Observables are mappings $A$:$Z \rightarrow Z$, which means, that an observable associates to every state $z$ in $Z$ a state $A(z)$ in $Z$.
III. If $A$ and $B$ are observables, then $C = AB$, the composition of $A$ and $B$, is also an observable.
IV. In the set of observables there is a neutral element Neutr with $A$(Neutr) = (Neutr)$A$ = $A$.
V. There is an 'impossible' zero state ($o$). There is also a zero observable ($O$) whose set spec $O$ of possible outcomes of measurement always consists of the logical variable 'false' (spec $O = \{\text{false}\}$). Formally:
   (a) $O(z) = o$ for all $z$ in $Z$,
   (b) $A(o) = O$ for all observables,
   (c) $A\ O = O\ A = O$ for all observables.
VI. There are logical observables on which logical operations can be defined. These observables are called propositions. They allow us to say when a system is in a given state with certainty.

## Appendix B. Summary of the proof

Let the system $\Sigma$ be identified with the contents of $B_N$ for a fixed $N > 0$ and let the single particles (atoms or molecules) in $B_N$ be the subsystems of $\Sigma$. In $\Sigma$ let the notion of a spin-like quality as a measure of complexity be defined. In the simplest case this is done by looking at the couplings of spins of particles instead of the spins of single particles.

Let $\sigma_i$ denote the spin of the $i$th particle in $B_N$ which can only have values $\pm 1$ and let the product $J_{i_1,\ldots,i_m} \cdot \sigma_{i_1}, \ldots, \sigma_{i_m}$ symbolize the coupling of sets of $m$ spins, where $J_{i_1,\ldots,i_m}$ is a number which determines the strength of the couplings. For instance

$$J_{i_1,\ldots,i_m} \cdot \sigma_{i_1}, \ldots, \sigma_{i_m} = \Sigma_{ij} J_{ij} \sigma_i \sigma_j$$

specifies the sum of products of pairwise couplings. This sum denotes the potential energy in statistical physics and is therefore a measure of complexity. Sums of triples, quadruples,…, $m$-tuples can be defined analogously. Clearly, independence of spins $\sigma_{i_1}, \ldots, \sigma_{i_m}$ is expressed by $J_{i_1,\ldots,i_m} = 0$ Within this notation the set

$$\{J_{i_1,\ldots,i_m} \cdot \sigma_{i_1}, \ldots, \sigma_{i_m} |\ \text{all permutations of indices } \{i_1, \ldots, i_m\}\}$$

is defined as to be the set $Z$ of states $z$ of the system $B_N$, where $m$ runs through the number of particles in $B_N$ . The zero state $o$ which has to be 'impossible' must be the one with no coupling at all. The set $Z$ of states of the system $B_N$ describes the different degrees of complexity of systems of spins covering $B_N$.

Let the set of observables A be defined as the set of all continuous deformations

$$f(J_{i_1,\ldots,i_m} \cdot \sigma_{i_1} \ldots \sigma_{i_m})$$

of the above spin configurations where $f$ is a continuous function. For instance ($J_{i_1,\ldots,i_m} \cdot \sigma_{i_1}, \ldots, \sigma_{i_m} + h\Sigma\sigma_i$) is a possible observable of the system with $h\ \Sigma\sigma_i$ describing the influence of an external field on the single spins with field strength $h$ arbitrarily small.

As, at least in principle, it is possible to define sets of outcomes of measurements of the observables, axiom I of weak quantum theory is satisfied. Axiom II, $A$ being a map $Z \rightarrow Z$, is satisfied since continuous deformations neither add nor delete spins.

Compositions of observables $A$, $B$ can be written as

$$g(f(J_{i_1,\ldots,i_m} \cdot \sigma_{i_1}, \ldots, \sigma_{i_m})) = g(K_{i_1,\ldots,i_m} \cdot \sigma_{i_1}, \ldots, \sigma_{i_m})$$

where $J_{i_1,\ldots,i_m}$ has changed to $K_{i_1}, \ldots, i_m$ under the influence of $f$ and where $f$ and $g$ are continous functions. The application of $g$ yields a state, too. Therefore Axiom III is satisfied. Since in general

$$g(f(J_{i_1}, \ldots, i_m \cdot \sigma_{i_1}, \ldots, \sigma_{i_m})) \neq f(g(J_{i_1,\ldots,i_m} \cdot \sigma_{i_1}, \ldots, \sigma_{i_m}))$$

compositions are not commutative.

As the Identity Id is a unique continuous function, a neutral observable Neutr = Id which does not deform any of the configurations (axiom IV) is also available.

Following axiom V, the application of the zero-observable $O$ to every state $z$ has to result in the 'impossible' zero state $O$ (ie $O(z) = o$) which is the one with no coupling at all. Therefore, the continuous deformation which corresponds to the zero observable must be represented by the function which erases any coupling of the spin configuration. Since the deformations have to be continous this will never occur and the set spec $O$ will always consist of the logical value 'false'. Since unstructured sets of spins are not restructured again by continuous deformations, we have $A(o) = o$ for any deformation $A$ applied to the zero state $o$. By $O(z) = o$ and $A(o) = o$ the continuous deformation $AO$ of $O$ results in $AO = O$. As $AO(z) = A(o) = o = O(z) = AO(z)$, $A$ and $O$ commute required by axiom $V$.